The unusual projection for one of John Dee's maps of 1580

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A brief description is given of the cartographic activity of John Dee, which involved the production of at least three manuscript maps in the years 1580-1583 in connection with exploration and imperial ambitions. The form of an idealised version of the unusual projection used in one of the maps dated 1580 is discussed and it is shown how it may be considered to be related to projections developed between antiquity and 1551. The precise form of Dee's projection is investigated and the possible reasons why he adopted it are considered.

Keywords: map projections, manuscript map, John Dee

Introduction

The scholar John Dee (1527-1608) is often thought of as Elizabethan England's greatest magus, but he has also been described by William Sherman (1998) as the English court's leading imperial geographer. Articles by a number of other authors have also mentioned his importance in this area (e.g. Taylor, 1930; De Smet, 1973; Macmillan, 2003). De Smet says that 'Il est impossible de déterminer exactement l'activité géographique et cartographique' of Dee, but in this capacity he was certainly responsible for the production of several maps (De Smet, 1973, p.111).

There are two extant manuscript maps that are definitely attributable to him, and probably a third, but there are no known printed maps. The reason for non-publication may, at least in part, be the fact that the known maps were concerned with matters of national importance - the possibility of finding northeast or northwest passages or claims to territory. MacMillan (2003) has discussed extensively the measures taken at the time by the state to retain the advantages that secrecy about recent geographical discoveries provided. One of the extant manuscript maps is of the western half of the northern hemisphere and is dated 1580 (British Library, MS Cotton Augustus I.i.1). It is this map that forms the major interest of the present paper and it is shown in Figure 1.

Whether Dee actually drew any of the maps himself is not certain. Ganong (1937) expressed the opinion that at least as far as the map of Figure 1 is concerned, it seems unlikely that Dee had the skill and technique to compile and draw such a map and suggested that it was probably made under his supervision. Whatever may be the truth of this, it seems very likely that even if someone else drew the map, Dee, with his mathematical knowledge and with his contacts with Gemma Frisius, Oronce Fine, Mercator, Ortelius and Nuñez (Taylor, 1930, pp 78-88; MacMillan, 2001), was responsible for the projection used.[1] Before this map and its projection are discussed in detail, the other two maps, which are drawn to well-known projections, are described very briefly. For details of the origins, purposes and political importance of all the maps and for illustrations of those not shown here see, for example, Sherman (1998) and references therein.

Dee maps drawn to well-known projections

One of these maps, which is signed by Dee, is another map of part of the northern hemisphere, also produced in 1580. It is now bound into Lord Burghley's copy of Ortelius's *Theatrum Orbis Terrarum* and is in the Burghley House Collection (number BKS16611). The map is described by Sherman (1988) as being on a 'fan-shaped projection' and it looks at first like an equidistant conic projection. In fact it represents 210° of longitude of a polar equidistant azimuthal projection, covering latitudes from 40° degrees north to the pole. Meridians are labelled from 10° to 200° and the 20° meridian appears to pass through London, so that the coverage in modern terms is notionally from 30°W to 180°E. The purpose of the map was to illustrate a possible northeast passage.

The second map is of the whole of the northern hemisphere. It was drawn on a polar stereographic projection in about 1583. It is often called the 'Humphrey Gilbert map' because it bears in the bottom right corner the inscription 'Humfray Gilbert Knight his charte', but it is thought to be by Dee (Sherman, 1998, pp 6-8, particularly note 27). Cormack (1997, p.99) implies that the projection used was Dee's invention[2]. It is, however, certain that Dee was not the inventor of this projection, because it was known to Hipparchus, Ptolemy and probably earlier to the Egyptians. The first extant world map on this projection was produced in 1507 by Gualterius Lud and it was also used well before Dee's time by Reisch, Apian and others (Snyder, 1997, p.22). It is, however, possible that Dee was unaware of the earlier use and reinvented the projection, but this seems unlikely because of his friendship, already mentioned, with cartographers such as Mercator and Ortelius.

The unusual projection

Dee's map of 1580 that is the subject of this paper is sometimes said to be 'of the northern hemisphere' but actually it is a map of the north-western semi-hemisphere, i.e. it attempts to depict only one quarter, not one half, of the earth's surface. The projection in the complete form used for this map is not described in any of the standard works on the history of map projections (Nordenskiöld, 1889, Keuning, 1955, Snyder 1993) . It may in fact be unique to this map and as far as I am aware it has not been commented upon previously by those who have discussed other aspects of the map.

The projection is based on an octant of the sphere centred at 35°16'N and approximately 80°W. The equilateral spherical triangle defining the octant is represented by three circular arcs of the same radius, one centred at the N pole and representing the equator and the other two representing the meridians at plus and minus 45° from the central meridian. Each of these two arcs is centred on the point where the other 45° meridian crosses the equator and they are drawn in bolder lines than the other parts of the graticule. The three arcs form a Reuleaux triangle.

The parallels are equally-spaced concentric circles centred at the pole and in the ideal form of the projection the meridians, other than those bounding the central octant, would be drawn so that they were equally spaced along the parallels within the central octant. This equal spacing would then be continued out beyond the bounding meridians of the octant to reach 90° either side of the central meridian, so enabling a full half-hemisphere to be depicted. The meridians so defined are not exactly circular except for those bounding the octant and their approximation by circular arcs is described later. A more technical description of the projection in its ideal form is as follows:

Choose axes centred at the N pole, O, with *y* vertical upwards and *x* horizontal to the right. The polar angle θ is measured clockwise from O*y*. If *R* is the (scaled) radius of the earth, the equation for the equally spaced lines of latitude ϕ, which are circles centred at the N pole, is $r = R(\pi/2 - \phi)$ and the equation for all the lines of longitude is $r = \pi R \cos[(\pi - \theta)/f + \pi/6]$, where *f* ranges from -2 at the meridian 90°W of the central meridian to +2 at the meridian 90°E of the central meridian. This parameter takes the values -1 and +1 for the two meridians bounding the central octant, which are circular arcs, unlike all the other meridians. The quantity *f* is related to the longitude λ east of the central meridian by $f = 4\lambda/\pi$. In cartesian coordinates, $y = r\cos\theta$, $x = r\sin\theta$.

The projection is neither equal area nor conformal (orthomorphic). The scale along the central meridian is uniform and the representative fraction is $1 : R/R_e$, where R_e is the radius of the earth. The scale is constant along each parallel. Setting $A = 1 - 2\phi/\pi$, the scale factor is equal to $A[2\cos^{-1}(A/2)-\pi/3]/\cos\phi$, which varies from $\pi/3=1.047$ for $\phi = 0$ to 4/3 = 1.333 for $\phi = \pi/2$. The projection is illustrated in Figure 2, with Tissot ellipses shown for a few selected points. These ellipses show the distortion on the map of imaginary infinitesimal circles on the ground at the centres of the ellipses (Snyder, 1993, pp 147-9 and references therein).

Relationship to other projections

The projection may be considered to be related to the series of equal-area pseudo-conic projections going back in origin to Ptolemy's second projection, the so-called homeotheric projection, and to the series of true geometric projections, Figure 3.

Ptolemy's projection (2nd century AD) was designed to cover only the then-known world, all of which lay between latitudes 16°25′S and 63°N (Nordenskiöld, 1889, pp 6 and 86; Keuning 1955, p.10; Snyder, 1997, pp 12-14; Berggren and Jones, 2000, pp 88-93). It was based on equidistant concentric circles for parallels, with all meridians, drawn from -90° to +90° from the central meridian, ideally cutting the parallels at their correct proportional distances from the central meridian along each parallel. In practice the meridians were approximated by circular arcs passing through the correct points on the extreme and central parallels, so that the projection is then not quite equal-area or homeotheric. The common centre of the parallel circles is chosen to give the impression that the viewer is looking along the straight line that joins the central point of the projection, i.e. the intersection of the central meridian and central parallel, to the centre of the earth. The equatorial diameter of the earth perpendicular to the line of sight would then appear to be tangent to the central parallel. With Ptolemy's choice for the length of this central diameter as equivalent to 180° at the latitude scale of the central meridian and with the central parallel at 23°50′N (the latitude of Syene) the centre of the circular arcs of the parallels is calculated to be at a distance equivalent to 181°48' of latitude north of the equator[3].

Sylvano's projection for his 'world map', which is of the type later described by the name of Bonne, is similar to the homeotheric projection but places the centre of the equally-spaced circular arcs of the parallels only about 100° equivalent from the equator. It also extends the cover to 160° either side of the central meridian, using correctly spaced non-circular meridians (Sylvano, 1511; Nordenskiöld, 1889, plate 33; Snyder 1993, pp 33-4). This projection is sometimes called a cordiform projection but was not extended by Sylvano beyond 40°S, so that the tendency to a point at the S pole is not seen.

The true cordiform projections are those of Stabius, of about 1500, further publicised by Werner in 1514 (Nordenskiöld, 1889, p.88; Keuning, 1955, pp 11-13 and Figs 10 & 11; Kish, 1965; Snyder 1993, pp 33-8). The three Stabius/Werner projections all have the N pole as the centre of equally spaced circular arcs representing the parallels and the meridians are equally spaced along all parallels by amounts correctly proportional to the corresponding equatorial spacings. The differences between the three projections lie in the different ratios of the distances representing one equatorial degree to those representing one degree of latitude along the central meridian. For projections 1, 2 and 3 these ratios are $\pi/2$, 1.0 and $\pi/3$, respectively. This ratio for Stabius/ Werner 3 is thus the same as for Dee's projection, and means that the N pole and the points where the meridians at ±45° from the central meridian cross the equator form an equilateral triangle in this projection, as they do in Dee's projection. The first of the three projections can be used for a longitude range of about 229°, whereas the other two can be used for the whole world.

The equilateral triangle representation of the N pole and the points on the equator at ±45° from the central meridian is also found for the three well-known true perspective projections shown in Figure 3, provided that the projection plane is tangent to the earth at the point 35°16'N on the central meridian and the axis of projection is normal to the plane. Their projection points are, respectively, 'infinity', the point at 35°16'S on the meridian opposite to the central meridian, and the centre of the earth. Only the first two of these can, however, be used to map a full 180° of longitude. Figure 4. shows a comparison of the chief features of Dee's projection with those of Stabius/Werner 3 and the orthographic and stereographic projections drawn at such scales that the equilateral triangles at their centres are of equal sizes.

The remaining two projections shown in Figure 3, the da Vinci octants and the Fine octant are based simply on Reuleaux triangles. Leonardo da Vinci or, as suggested by Keuning (1955), one of his pupils, drew a world map (c.1514) consisting of eight octants arranged in two groups of four, one group for the northern hemisphere and one for the southern hemisphere with each group arranged in a four-leaf clover pattern (Nordenskiöld, 1889, pp 76-7; Keuning, 1955, pp 22-3; Snyder 1993, p.40). Figure 5 shows a modern reconstruction of this type of projection by Furuti[4], although the da Vinci map octants do not have graticules. The final projection shown in Figure 3 is that of Oronce Fine. A graticule for a single Reuleaux triangle was given by Fine in his *Sphaera Mundi* (1551) but not used by him to produce a map. The projection is described and illustrated by Nordenskiöld (1889, p.94), by Keuning (1955, p.23 and fig 20) and by Snyder (1993, p.40-1). The parallels are circular arcs equally spaced and centred at the pole. The equatorial arc and the arcs representing latitude 45° are each divided equally for degrees of longitude and the meridians are drawn as circular arcs passing through the pole and the appropriate points on the equator and on the 45° parallel. Keuning (1955, p.23) records only one map drawn using this projection. Dee's map is an extension of this type of projection to cover 180° of longitude.

Various kinds of octant maps have been revived in the twentieth century but have not generally been taken seriously (Snyder, 1993, pp 264-5).

The form of Dee's meridians

In order to examine whether Dee's graticule was drawn using the ideal form of the meridians or the circular form suggested by Fine, a digitised version of the map was obtained by scanning Figure 1 of Sherman's paper (1998). It was considered that the further manipulation and attempted flattening of the original map to obtain a direct digital scan would not be justified with only this aim in mind, nor was it likely that a much better image would be obtained. The results to be described justify this decision[5].

The intersections of meridians and parallels were calculated at ten degree intervals of latitude and longitude for each assumed form of the meridians. The intersections of the ±45° meridians with the parallels were also included. There are 190 intersections of parallels and meridians on each calculated graticule and it was possible to locate 166 (87%) of these on the scanned Dee map. Examination of the original map showed that three of the intersections would lie outside the area covered by it, these being the equatorial points at longitudes ±90°and the point at latitude 10°, longitude 90°E of the central meridian. Five other equatorial points were not locatable on the scan, as were a number of points on the meridians at ±70° and ±80°. At least three of these 21 points would have been difficult to locate accurately on the original or a direct scan of it because they were either too feint or obscured by deterioration of the map.

The 166 points of the Dee graticule were digitised twice independently in order to see how accurately this could be done. The standard deviation between the two sets of locations obtained was equivalent to approximately 0.4 mm measured on the original map. The average of the two sets was used in the comparison with the two calculated graticules, in the following way.

For each calculated graticule, only those intersections lying on the Reuleaux triangle and the central meridian of the Dee graticule were effectively shifted in the horizontal and vertical directions, scaled and rotated to obtain a best fit between them and the corresponding points of the calculated graticule. Best fit was defined as that giving the least value of the sum of the squares of the remaining differences in positions, i.e. a bi-dimensional regression with five parameters was performed. As expected, the parameters for both fits were found to be almost identical, and the scaling factors in the horizontal and vertical directions were found to differ by only 0.04%. Using the parameters found, the standard deviations were calculated for all the 166 points of intersection, for the 94 points within and including the Reuleaux triangle and for the 73 intersections lying outside the Reuleaux triangle (the point at the pole was used in both of the last two calculations).

The standard deviations for the Reuleaux triangle, the 166 points, the 94 points and the 73 points, respectively, were 1.01, 2.0, 0.96 and 2.8 for the ideal meridians and 1.01, 1.6, 0.95 and 2.2 for the circular meridians, with all values quoted in effective mm on the original map. The corresponding deviations are shown multiplied by 5 in figure 6. These results show that it is not possible to distinguish between the two forms of the meridians within the Reuleaux triangle but suggest that at least outside this region the meridians actually used are more likely to have been circular than a good approximation to the ideal meridians. Figure 6 shows that there are systematic deviations from both fits, and these may be due, at least in part, to distortions of the parchment of the map since drawing or to difficulties in flattening it. Despite this, the accuracy of drawing is very high in the central region of the map (on or within the Reuleaux triangle), where the greatest shift of an intersection from its expected position corresponds to less than 2.5 mm on the original and the standard deviation, which corresponds to the most probable shift, is less than 1 mm.[6] The 'remarkable scientific accuracy [of this map] for the time' was commented on by MacMillan (2003), but without supporting measurements.

Dee's choice of projection

Why did Dee choose such an unusual projection? The clear focus of the map is North America or, as Dee preferred to call it, *Atlantis*. The projection has the advantage, shared with the true perspective projections, if their plane of projection is correctly chosen, of giving the observer the impression that he is looking at this central feature directly from above. Almost the whole of the then-known part of North America falls within the Reuleaux triangle, and this is the area that was of principal interest to the English state, as is clear from the inscription on the reverse of the map describing English North Atlantic voyages.

Dee was, however, clearly concerned to show in addition the correct relationship between North America and Europe to the east and Japan to the west. He therefore required a projection that did not distort longitudinal distances in the way that all the true perspective projections do, as shown for the orthographic and stereographic projections in Fig. 4. The only other projection of Fig 3 that comes close to satisfying these two advantages is Stabius/Werner 3. According to Keuning (1955, p.12), Stabius/Werner 3 was used by Fine for a world map in 1536, by Giovanni Paolo in 1556 and by Hadji Ahmed in 1560. Fig. 4 shows that this projection differs only slightly from Dee's projection except for the regions towards ±90° from the central meridian. It may be, however, that Dee was unaware of Stabius/Werner 3, although this seems unlikely because of his close connections with well-known cartographers.

Two facts may have influenced his choice. The first is that the basis of his projection is the octant projection suggested by Fine, described above but taken up by only two cartographers. Taylor (1930, p.86) has described Fine as 'the fourth of Dee's great teachers'. Dee was his pupil in Paris during the years 1550 and 1551, and it was during the latter year that Fine published his octant projection; it is therefore possible that Dee had discussed it with him. Another fact that Dee may have taken into account is the geometrical nicety that the Stabius/Werner projection does not treat the bounding great-circle arcs of the central octant in a precisely equal manner, whereas Fine's and Dee's projections do.

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Figures

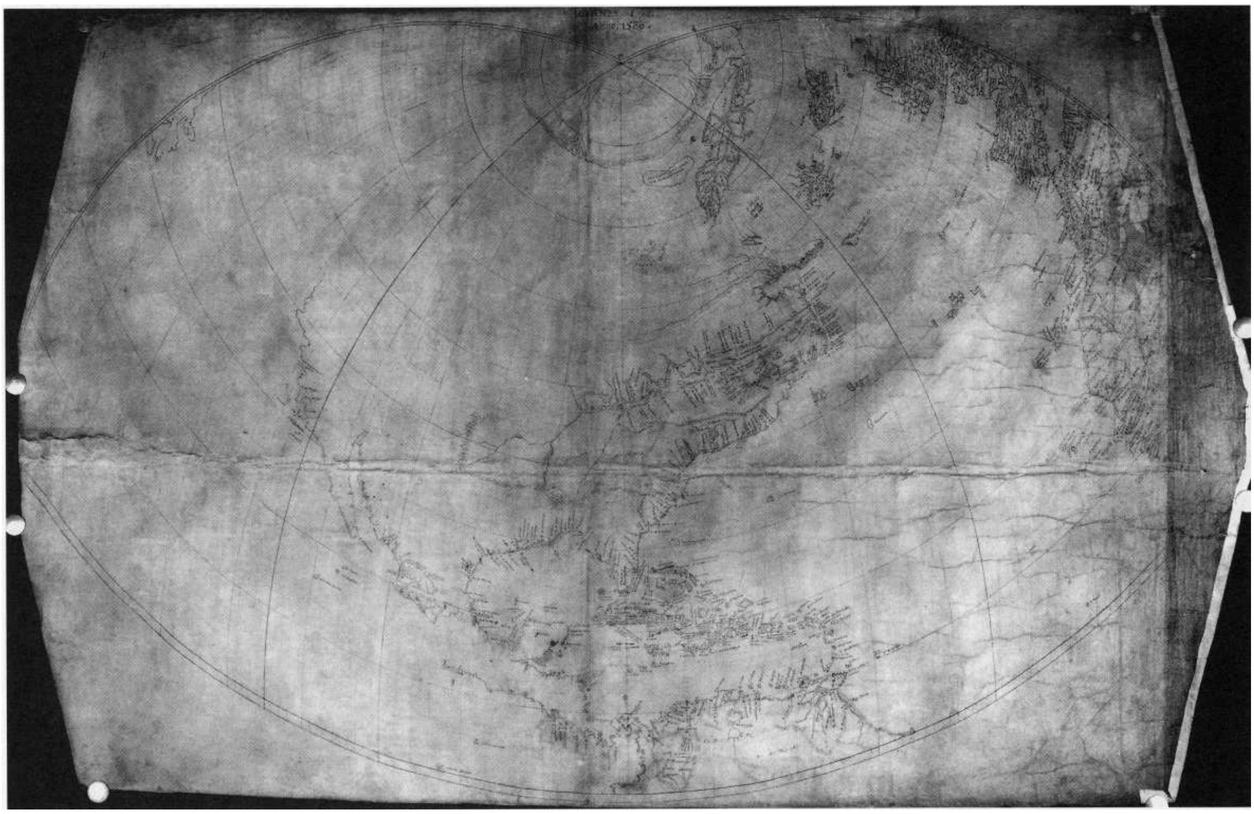

Fig.1. John Dee's map of the north-western semi-hemisphere, dated 1580. Cotton Augustus I.i.1;

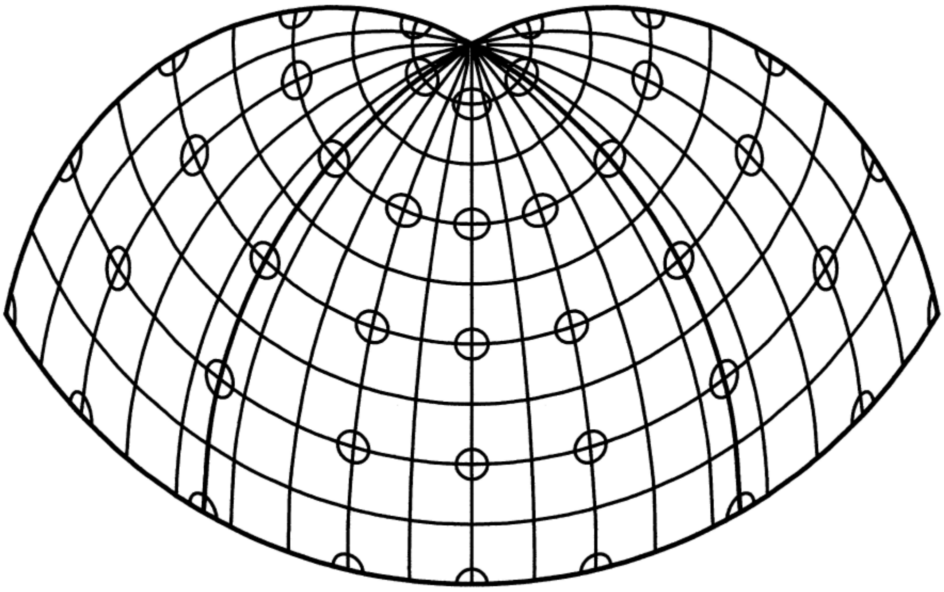

Fig.2. Reconstruction of the projection used by John Dee for the map shown in Figure 1. The reconstruction was made using the equation $r = \pi R\cos[(\pi - \theta)/f + \pi/6]$ for the meridians, but at the scale shown the use of circles as described in the text would be barely distinguishable.

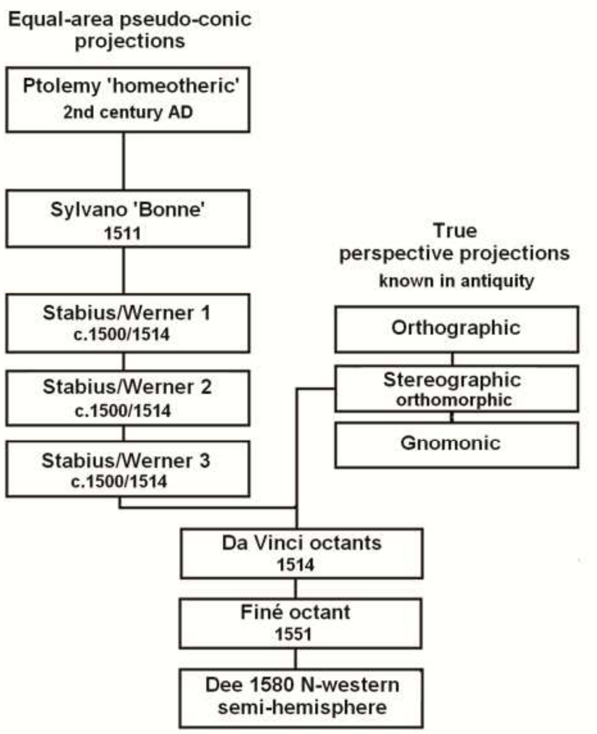

Fig.3. Schematic representation of the relationship between Dee's projection and earlier projections

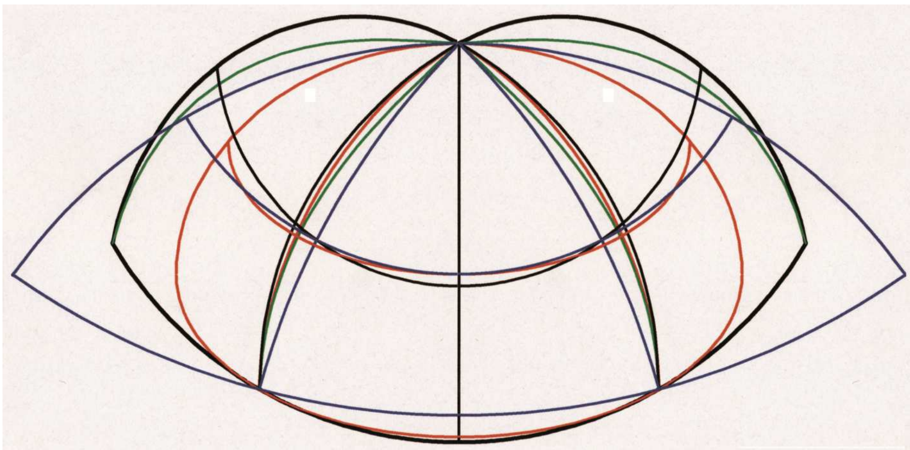

Fig.4. Comparison of Dee's projection with other projections for which the points at 90°N and at 0°N45°E and 0°N45°W of a central meridian lie at the vertices of an equilateral triangle. Only the equator, the parallel at 35°16'N, the central meridian and the meridians at ±45° and ±90° from the central meridian are shown. The Dee projection is shown in black, the orthographic in red, the stereographic in blue and the Stabius/Werner 3 in green. For the latter, all parallels coincide with those of the Dee projection

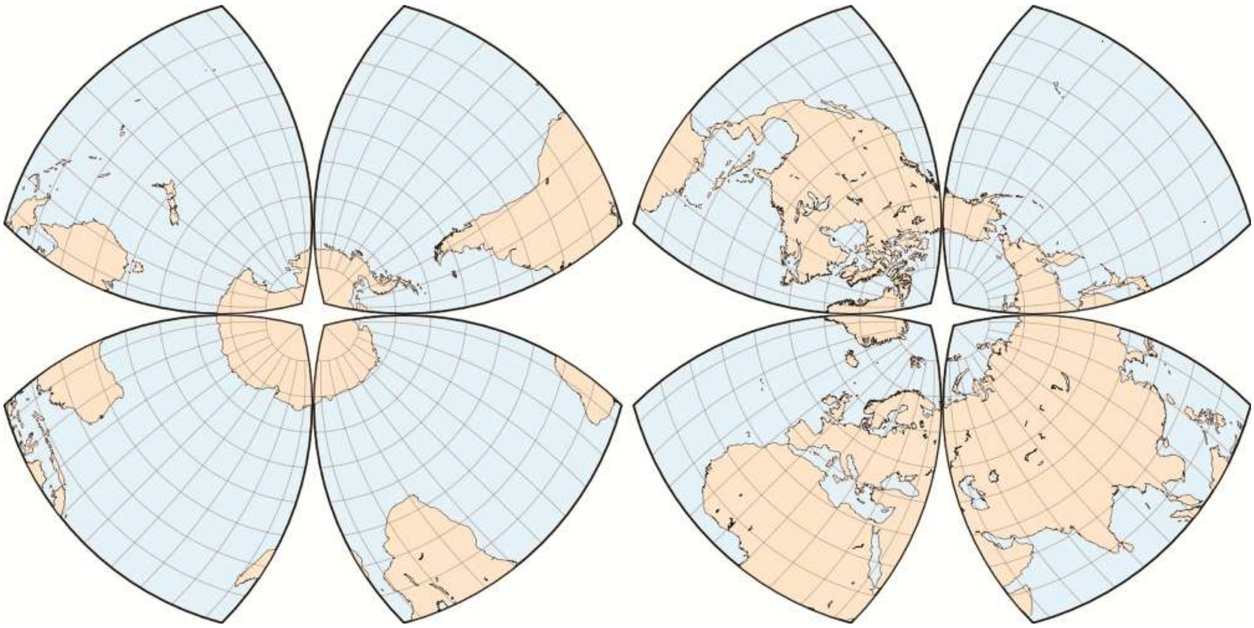

Fig.5. Furuti's reconstruction of the Da Vinci map in octants. The parallels are circles through points correctly placed on the central and extreme meridians of each octant and the meridians are equally spaced along the parallels.

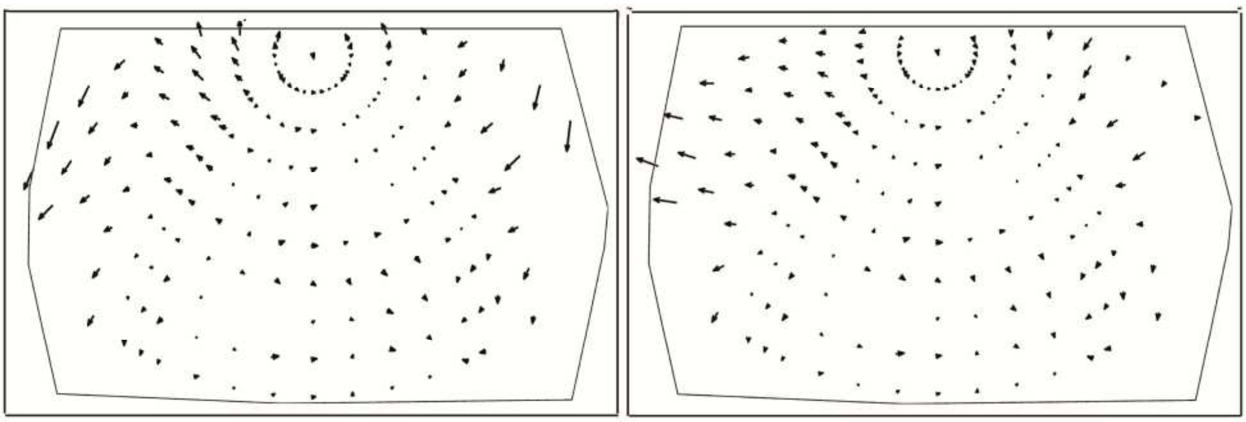

Fig.6. Deviations from fits. The left-hand figure relates to the comparison of Dee's graticule with the graticule calculated for ideal meridians and the right-hand figure shows the deviations in the same way for the comparison with the graticule calculated for circular meridians. In each case a best fit was made only to the points of the Reuleaux triangle and the central meridian. The tails of the arrows indicating the displacements represent the positions in Dee's map and the displacements required to bring Dee's graticule into coincidence with the calculated graticules are indicated by the lengths of the arrows, which are exaggerated by a factor of 5 compared with the scales of the graticules.

References

Cormack, L. B. (1997) **Charting an Empire: Geography at the English Universities 1580-1620,** University of Chicago Press, Chicago and London, p.99.

De Smet, A. (1973) ‘John Dee et sa place dans l’histoire de la cartographie’, in **My Head is a Map: Essays and Memoirs in Honour of R. V. Tooley**, Ed. Helen Wallis and Sarah Tyacke, Francis Edwards and Carta Press, London

Fine, Oronce (1551or 1555), **De Mundi sphaera, sive cosmographia libri 5**, Paris

Ganong,. W. E. (1937) ‘Crucial maps in the Early Cartography and Place-Nomenclature of the Atlantic Coast of Canada: IX’, **Transactions of the Royal Society of Canada**, XXXI 101-30. Reprinted in **Crucial Maps**…with an Introduction, Commentary, and Map Notes by Theodore E. Layng., University of Toronto Press and Royal Society of Canada, 1964

Keuning, J. (1955) ‘The history of geographical map projections until 1600, **Imago Mundi** 12, 1-24

Kish, G. (1965) ‘The Cosmographic Heart: Cordiform Maps of the 16th Century’, **Imago Mundi**, 19, pp. 13-21.

MacMillan, K. (2001) ‘Discourse on History, Geography, and Law: John Dee and the Limits of the British Empire, 1576-80’ **Canadian Journal of History** 36, 1-25

MacMillan, K. (2003) ‘Sovereignty “More plainly described”:Early English Maps of North America, 1580–1625’ **Journal of British Studies** 42, 413-47

Nordenskiöld, A. E. (1889). **Facsimile Atlas to the Early History of Cartography with Reproductions of the Most Important Maps Printed in the XV and XVI Centuries**. English translation of Swedish original by J. A. Ekelöf and C.R. Markham, Stockholm,1889, reprinted by Dover Publications, New York, 1973

Sherman, W. H. (1998) ‘Putting the British Seas on the Map: John Dee’s Imperial Cartography’, **Cartographica**, 35 (3/4): 1-10.

Snyder, J.P. (1993) **Flattening the Earth. Two Thousand Years of Map Projections**, (University of Chicago Press, Chicago and London, paperback edition 1997

Sylvano, B. (1511) **Ptolemy’s Geography (Claudii Ptholemaei Aleandrini liber geographiae, cvm tabvlis et vniversali figvra et cvm additione locorum quae a recentioribvs reperta svnt, diligenti cvra emendatvs et impressvs)**, J. Pentius de Leucho,Venice, Jacobus Angelus version, edited and remodelled by Sylvano. p.153 (not present in all copies).

Taylor, E. G. R. (1930). **Tudor Geography 1485-1583**, Methuen & Co. Ltd, London, pp. 75-139

Notes

[1] As Macmillan (2001) points out, Dee had the works of most of the well-known cartographers of the day, as well as of ancient cartographers such as Ptolemy, in his personal library, which is catalogued in R.J. Roberts and A. G. Watson’s *John Dee’s Library Catalogue* (London, 1990).

[2] Cormack also suggests that this projection is linked to Dee’s ‘Paradoxal Compass’ and R. Baldwin discusses this question further in ‘John Dee’s interest…’. in *John Dee: Interdisciplinary studies in English Renaissance Thought* Ed Stephen Clucas (Springer, Dordrecht c. 2006) p.100. See also Appendix A to *A Regiment for the Sea,* by William Bourne, Ed E.G.R.Taylor, (Hakluyt Society, 2nd Ser. CXXI , CUP, 1963), pp 415-7

[3] The true central parallel according to Ptolemy’s intended latitude range is actually about half a degree S of the parallel of Syene. Also, the projection does not fulfil Ptolemy’s intention that the chord of the equatorial arc be tangential to the arc for the parallel of Syene.This is because the 90° meridional arc is taken to be at a distance equivalent to half the originally assumed equatorial chord length along the equatorial arc from the central meridian. This does not put the meridian at the end of the originally assumed equatorial chord. A more consistent approach would be to make the total length of the equatorial arc equal to 180° at the same scale as that chosen for the central meridian. The equatorial chord length then becomes equivalent to 171°17' and the radius of the equatorial arc becomes equivalent to 165°48' rather than Ptolemy’s 181°48'.

[4] I am grateful to Carlos Furuti for permission to use this figure, which appears on his web site http://www.progonos.com/furuti/MapProj/Normal/ProjPM/projPM.html, last accessed 23 March 2011

[5] The map measures 68x105 cm, is on parchment and is stored tightly rolled.

[6] It is assumed that the shifts or errors r in position conform to a normal distribution $P(r) = A\exp\left[-r^2/(2\sigma^2)\right]$ with standard deviation σ. A is a constant and $P(r)$ is the probability per unit area of finding a displacement r from the correct position.